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# Aircraft non-volatile particle emissions: estimating number from mass

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## 1. INTRODUCTION

- Aircraft emissions of non-volatile particulate matter (PM) contribute to anthropogenic climate forcing and degrade air quality.
- Direct radiative forcing (RF) is proportional to emitted *mass* of particles.
- In-direct RF due to contrails and induced cloudiness is potentially greater than the direct RF, however it is highly uncertain [1].
- The *number* of ice particles in contrails has been shown to correspond to the *number* of non-volatile particles, affecting the contrail optical properties [2].
- Estimates of non-volatile PM mass emitted by aircraft have recently been revised [3].
- Estimating particle *number* emissions is required to accurately estimate global aviation climate impacts.

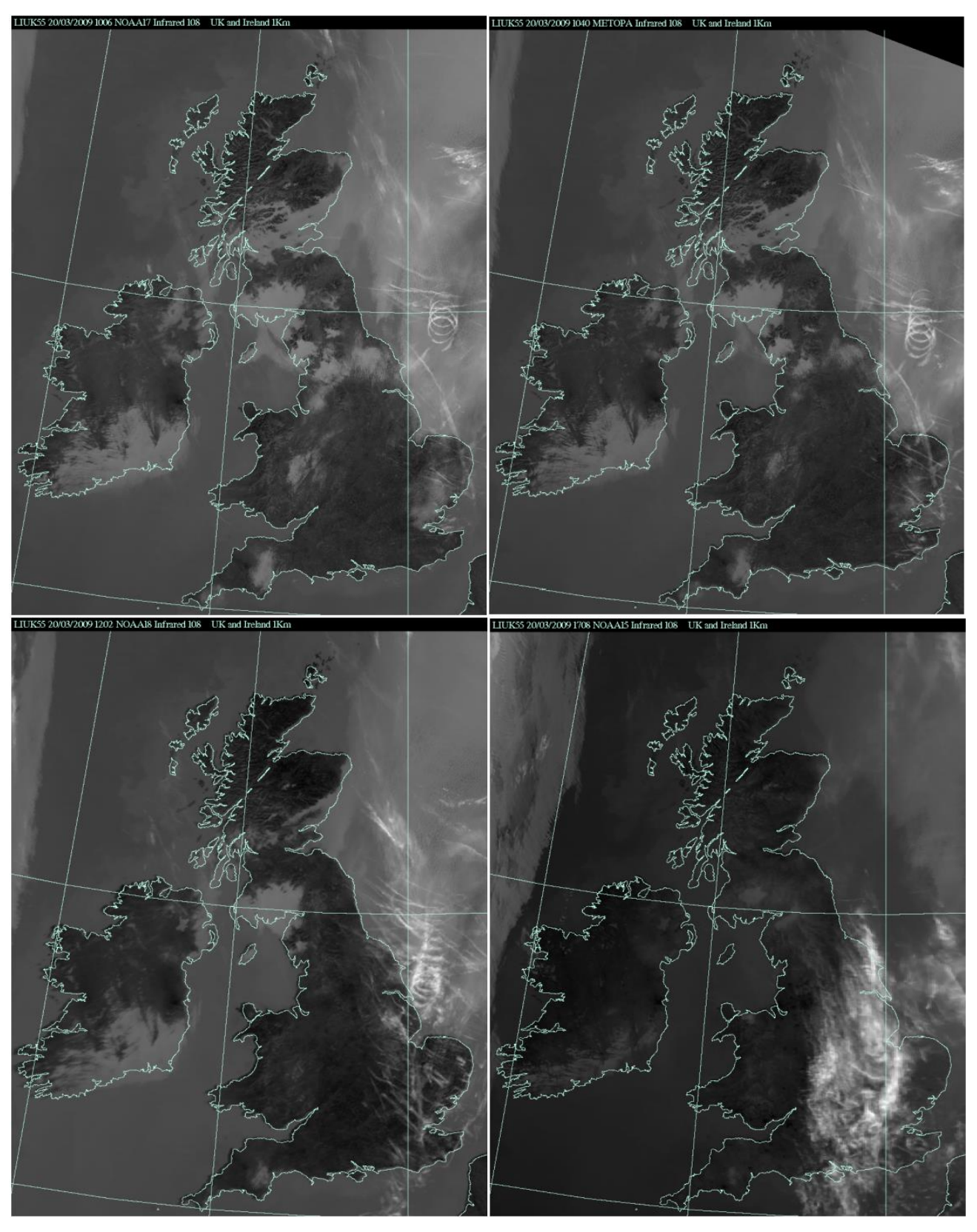

**Figure 1: Images of the formation of contrail induced cirrus over seven hours. Images published by Haywood et al. [4].**

## 2. THEORY

In the free molecular regime (Kn→∞) for diffusion limited cluster aggregation aggregates, the number, $n_{va}$, of primary particles of volume-surface equivalent diameter $d_{va}$ in an aggregate of mobility diameter $d_m$ is

$$n_{va} = k_a \left(\frac{d_m}{d_{va}}\right)^{D_m}, \qquad (1)$$

where $k_a \sim 1$ is a parameter describing agglomerate structure and $D_m$ is the mass mobility exponent [5,6].

The mass of an aggregate, $m$, is the sum of the mass of the primary particles and assuming a constant $d_{va}$ within a given aggregate,

$$m(d_{va}) = n_{va}\rho_0\left(\frac{\pi}{6}\right)d_{va}^3\,. \qquad (2)$$

For non-volatile PM emitted by aircraft engines, Boies et al. [7] have shown that $d_{va}$ is a function of $d_m$ and a power-law fit to experimental data yields $d_{va} = 0.79 d_m^{0.8}$ ($R^2$=0.86). For the same aggregates, Johnson et al. [8] have shown that $D_m = 2.76$. Combining equations (1) and (2) yields

$$m(d_m) = \rho_0\left(\frac{\pi}{6}\right)0.79^{(D_m-3)}d_m^{(2.4+0.2D_m)}\,, \qquad (3)$$

where $\rho_0 \sim 1900$ kg/m³ is the material density of soot comprised of elemental carbon. Assuming the number weighted aggregate mobility diameter distribution, $n(d_m)$, is represented by a mono-modal lognormal distribution, the total mass concentration $M$ is the $d_m^{(2.4+0.2D_m)}$–th moment,

$$M = \int_0^\infty m(d_m)n(d_m)\mathrm{dlog}d_m$$

$$M = \rho_0\left(\frac{\pi}{6}\right)0.79^{(3-D_m)}N_0\mathrm{GMD}^{\phi}\exp\left(\frac{\phi^2(\mathrm{logGSD})^2}{2}\right), \qquad (4)$$

where $N_0$ is the particle number concentration, GMD and GSD are the geometric mean and standard deviation of the distribution and $\phi = 2.4 + 0.2D_m$. Aircraft emissions are typically normalized to the mass of fuel burned, to give an emissions index. Re-arranging equation (4) gives

$$\mathrm{EI}_n = \frac{\mathrm{EI}_m}{\rho_0\left(\frac{\pi}{6}\right)0.79^{(3-D_m)}\mathrm{GMD}^{\phi}\exp\left(\frac{\phi^2(\mathrm{logGSD})^2}{2}\right)}, \qquad (5)$$

## 3. EXPERIMENTAL

**Ground level**
Measurements of aircraft engine (GE, CFM56-5B4-2P) PM emissions were conducted as part of the SAMPLE III campaign. Particle number and size distribution were measured using a DMS500 (Cambustion Ltd, UK). Non-volatile particle mass was measured via Laser Induced Incandescence (LII, Artium LII-300) [7].

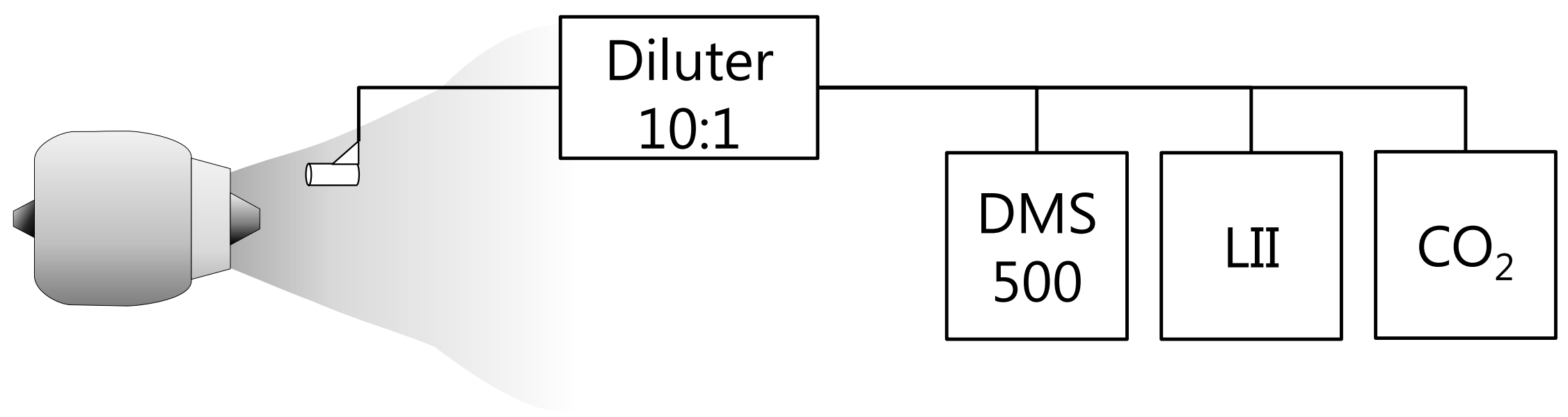

**Figure 2. Schematic of sampling system and instruments.**

**Cruise**
Measurements at cruise of particle mass, number and size distribution were conducted during the SULFUR 1-7 measurement campaigns [2].

**Emissions indices**
Engine emissions are normalised to fuel burn to give an emissions index, $\mathrm{EI}_m$ (g/kg-fuel) or $\mathrm{EI}_n$ (part./kg-fuel).

## 4. RESULTS - GROUND LEVEL

- Measured $\mathrm{EI}_n$ are compared to $\mathrm{EI}_n$ estimated using LII measurements of $\mathrm{EI}_m$ and the proposed method. Good agreement ($R^2$ = 0.98) is shown in Figure 2.
- For each data point, $\mathrm{EI}_n$, GMD and GSD were measured with a DMS500 and $\mathrm{EI}_m$ with an LII.

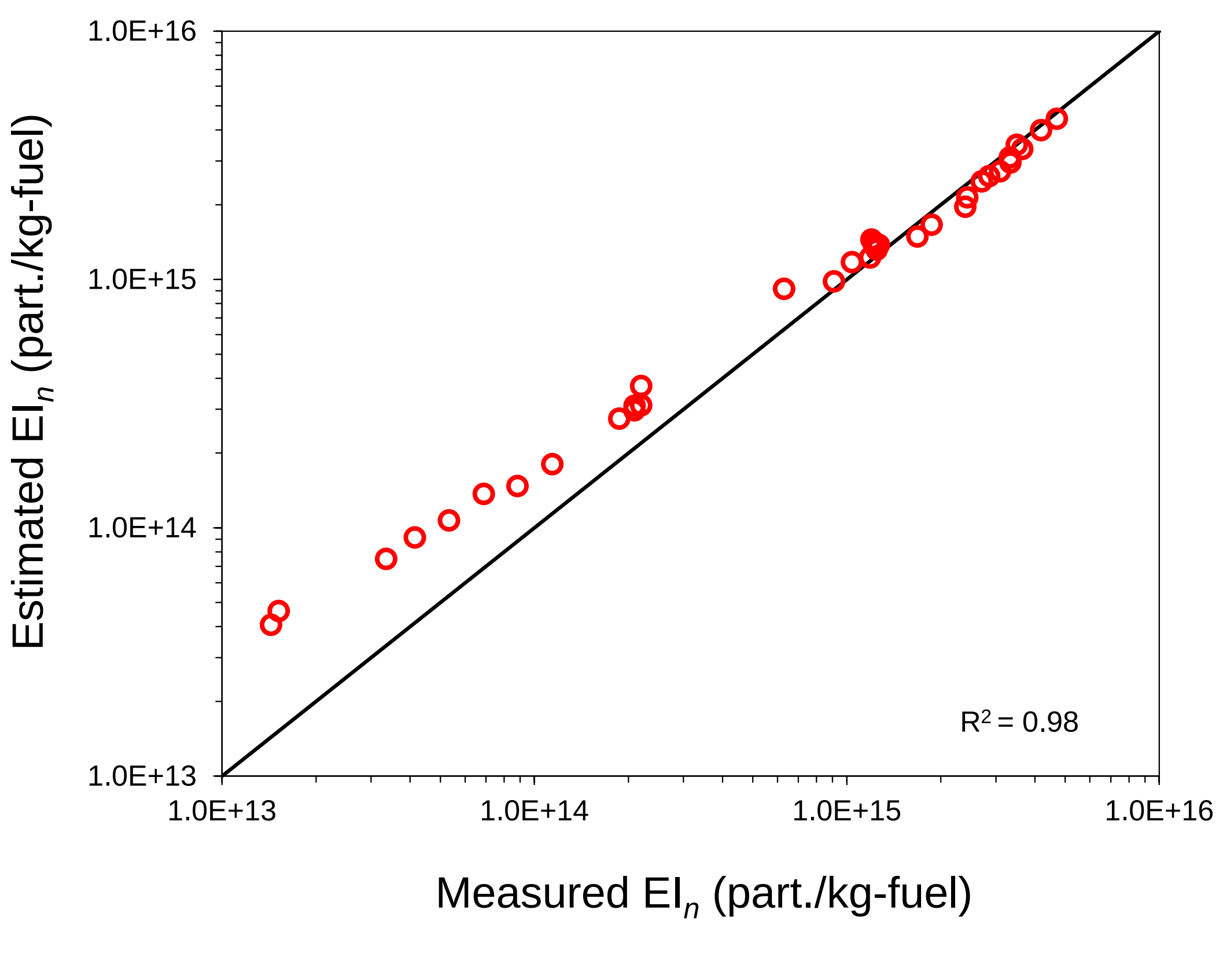

**Figure 2: Comparison between measured and estimated $\mathrm{EI}_n$ during ground level testing.**

## 5. RESULTS – CRUISE ALTITUDE

- Measured $\mathrm{EI}_n$ are compared to $\mathrm{EI}_n$ estimated using the proposed method and assumed GMD, GSD and $D_m$ shown in Table 1 from literature [2].
- Good agreement ($R^2$ = 0.98) is shown in Figure 3.

| | Measurements | | Assumptions | | | Estimate |
|---|---|---|---|---|---|---|
| Aircraft | $\mathrm{EI}_m$ (g/kg-fuel) | $\mathrm{EI}_n$ (×10¹⁵ part/kg-fuel) | GMD (nm) | GSD | $D_m$ | $\mathrm{EI}_n$ (×10¹⁵ part./kg-fuel) |
| B707 | 0.5 | 1.7 | 60 | 1.4 | 2.76 | 1.7 |
| ATTAS | 0.1 | 1.7 | 30 | 1.4 | 2.76 | 1.6 |
| A310 | 0.019 | 0.6 | 25 | 1.4 | 2.76 | 0.5 |
| B737 | 0.011 | 0.35 | 25 | 1.4 | 2.76 | 0.3 |
| A340 | 0.01 | 0.18 | 25 | 1.4 | 2.76 | 0.3 |

**Table 1: Aircraft PM measurements, assumptions and estimates at cruise.**

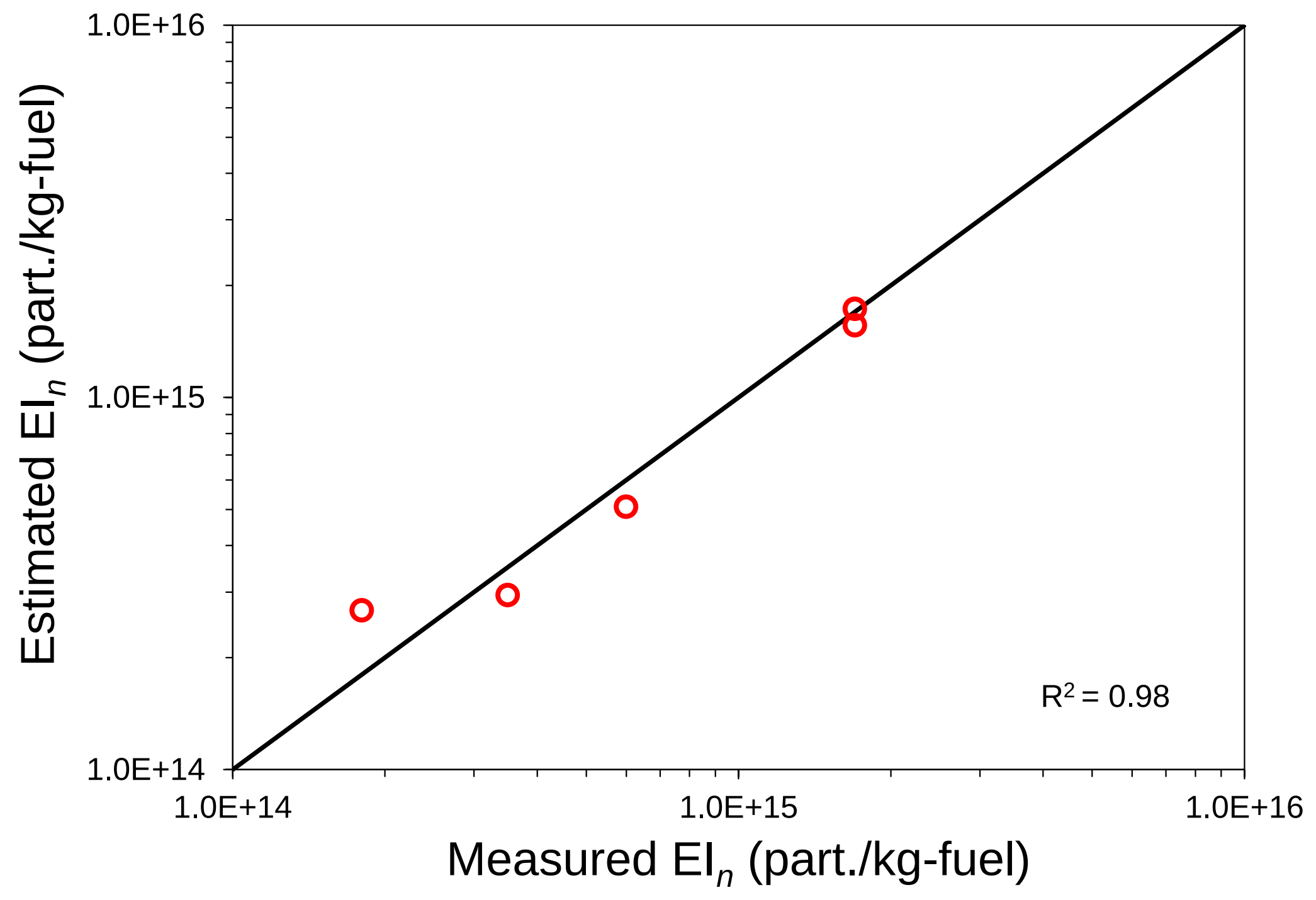

**Figure 3: PM measurements, assumptions and estimates at cruise.**

## 6. RESULTS - SENSITIVITY

- Global sensitivity indices [9] indicate that method is most sensitive to input GMD.

| Variable | Nominal value (range) | Sensitivity index |
|---|---|---|
| GMD | 25 (20-40) nm | 0.72 |
| GSD | 1.4 (1.3-1.5) | 0.03 |
| $D_m$ | 2.76 (2.5-2.9) | 0.29 |
| $\rho_0$ | 1900 (1800-1900) kg/m³ | 0.01 |

**Table 2: Global sensitivity indices of method to inputs.**

## 7. SUMMARY

- New method to estimate aircraft $\mathrm{EI}_n$ from $\mathrm{EI}_m$ proposed.
- Estimates show good agreement with measurements ($R^2$~0.98) at ground level and cruise.
- Sensitivity analysis indicates that GMD is the input parameter with greatest influence on estimates.
- Next steps are to:
  i. Develop relationship to estimate GMD as a function of engine operating point
  ii. Apply method to estimate global aircraft particle number emissions.

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